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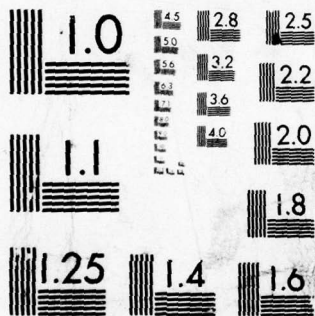
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AXIOMATIC SPECIFICATION  
OF  
SYNTAX-DIRECTED TRANSLATION

by

Sharon Sickel  
W.M. McKeeman

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## ABSTRACT

Predicate logic is applied to the specification of syntax-directed translation. Context-free grammars are shown to be representable by logic programs, and a translation from grammars to logic programs is presented as a logic program. Coupled grammars are introduced, shown to be interpretable as translators, shown to be representable as logic programs, and translation from coupled grammars to logic programs is presented via logic programs. Mixed grammars, a concisely representable special case of coupled grammars, are given in terms of coupled grammars.

### 1. Introduction

We define grammars to be coupled iff they are context-free and there is a 1-1 correspondence between their productions. Two different interpretations are given for them. The first is a generalization of the syntax-directed transductions of Lewis and Stearns [2]. Their productions take the form

$$A \rightarrow \alpha, \beta$$

where the parts of the form

$$A \rightarrow \alpha$$

constitute the underlying input grammar, and the string  $\beta$  specifies the output. The nonterminals of each  $\beta$  must be a permutation of those in the corresponding  $\alpha$ . As  $A \rightarrow \alpha$  is applied during the parse, the value of  $A$  is given by  $\beta$  where each nonterminal is replaced by its own value. Our paired productions take the form

$$A \rightarrow \alpha \quad \text{and} \quad A \rightarrow \beta$$

but we relax the requirement of a 1-1 correspondence between nonterminals in  $\alpha$  and  $\beta$ . A missing nonterminal in  $\beta$  is ignored, its value being discarded. An extra nonterminal in  $\beta$  is kept literally, standing in the output for any string derivable from it by the productions in the second grammar.

The second interpretation of coupled grammars links input to output via the parse

sequence. Aho and Ullman [1, Vol. II] present a similar scheme based on the production form

$$A \rightarrow \alpha, \beta$$

where again the nonterminals in  $\alpha$  and  $\beta$  must correspond. Our productions take the form

$$A \rightarrow \alpha \quad \text{and} \quad B \rightarrow \beta$$

where there are no restrictions on the form or correspondence of symbols within the productions.

The logic program corresponding to a grammar provides a static statement of the meaning of the grammar; that is, an axiomatic specification of it. Each predicate in the logic program specifies a relation among its arguments.

Mixed grammars are a concise notation for the simple syntax-directed translation schemata presented by Aho and Ullman [1, Vol. I]. There is a nonterminal vocabulary  $V_N$  and two terminal vocabularies,  $V_I$  for the input and  $V_O$  for output. Mixed grammars are a directly executable notation on a simple pushdown stack machine. They have a concise representation in logic. They are, furthermore, self-describing, and self-translating.

## 2. Logic Representations of Grammars

A context-free grammar  $G$  is a four-tuple  $[V_N, V_T, P, S]$  as usual.  $P$  is a set of productions of the form

$$A = \alpha$$

where we have used the symbol  $=$  instead of  $\rightarrow$  to avoid confusion with  $\rightarrow$  for implication.

$$A \in V_N \quad \text{and} \quad \alpha \in (V_N \cup V_T)^*.$$

The empty string is denoted by  $\epsilon$ . We assume a function,  $\text{start}(G) = S$ . We further



follow the conventions that nonterminals are denoted by capital letters and terminals by any other symbols except Greek letters which we sometimes use to denote strings. Because of these conventions and the existence of the function, start, we can completely specify a grammar as a sequence of productions, and we frequently do.

A logic program [4] is a set of WFs in the form

$$L \leftarrow R$$

and one WF, the call, of the form

$$\leftarrow R$$

where  $L$  is a predicate and  $R$  is a conjunction of predicates. All variables are implicitly universally quantified.

Table 2.1 is an example of a context-free grammar and a corresponding logic program. Terminal symbols in the grammar are delimited by double quotes, and the parameters to the predicates are strings of symbols represented by the juxtaposition of variable names and constants delimited by double quotes. We may drop the quotes when the meaning is obvious.

$G = RG$	$G(x1\ x2) \leftarrow R(x1) \wedge G(x2)$
$G = R$	$G(x1) \leftarrow R(x1)$
$R = L = "F"$	$R(x1\ "="\ x2) \leftarrow L(x1) \wedge F(x2)$
$F = PF$	$F(x1\ x2) \leftarrow P(x1) \wedge F(x2)$
$F = P$	$F(x1) \leftarrow P(x1)$
$P = ""U""$	$P(""x1\ """) \leftarrow U(x1)$
$P = L$	$P(x1) \leftarrow L(x1)$
$P = U$	$P(x1) \leftarrow U(x1)$
$L = "G"$	$L("G") \leftarrow$
$L = "R"$	$L("R") \leftarrow$
$L = "F"$	$L("F") \leftarrow$
$L = "P"$	$L("P") \leftarrow$
$L = "L"$	$L("L") \leftarrow$
$L = "U"$	$L("U") \leftarrow$
$U = L$	$U(x1) \leftarrow L(x1)$
$U = "="$	$U("=") \leftarrow$
$U = ""$	$U("") \leftarrow$

Table 2.1: Grammar GG and its logic program representation LPGG.

The interpretation of the grammar GG is obvious [3]. It is in fact self-describing. The correspondence between each production of grammar and Horn clause is straightforward. For example,

$$G(x_1 x_2) \leftarrow R(x_1) \wedge G(x_2)$$

states that if R is true on string  $x_1$  and G is true on string  $x_2$ , then G is necessarily true on the concatenation of  $x_1$  and  $x_2$ , which is also the meaning of

$$G = RG$$

in the grammar.

$$L("G") \leftarrow$$

states that L is always true for the letter G, and so on.<sup>§</sup>

### 3. Translation from Context-free Grammars to Their Logic Interpretations

Predicate TGL formally defines the relationship between context-free grammars and their interpreting logic programs, and also the translation process between the two, i.e.,  $TGL(G, LP)$  is true iff LP is the logic program representation of grammar G. In the example of Table 2.1,  $TGL(GG, LP_{GG})$ .

$$TGL(\epsilon, \epsilon) \leftarrow$$

$$TGL((A = \beta) \cdot G, \text{New\_rule} \cdot LP) \leftarrow$$

$$TGL(G, LP)$$

$$\wedge TPL(\beta, l, \text{Iparam}, \text{Conj})$$

$$\wedge \text{New\_Rule} = (A "(" \text{Iparam} ")" \leftarrow " \text{Conj})^{\ddagger}$$

---

<sup>§</sup> In examples we occasionally optimize logical formulas to remove the value true where it is formally present but contributes no meaning. In particular  $A \wedge \text{true} \Leftrightarrow A$  and  $(A \leftarrow \text{true}) \Leftrightarrow (A \leftarrow)$ .

<sup>‡</sup> The symbol "." appears frequently in parameters of logic programs.  $u \cdot v$  denotes a parameter that is subdivided into components  $u$  and  $v$ . For strings we denote such a decomposition simply by juxtaposition of symbols, e.g.  $uv$ . The use of "." denotes decomposition of more complicated structures, such as grammars. The first use here,  $(A = \beta) \cdot G$ , denotes a grammar that is subdivided into its first production  $A = \beta$  and the remaining productions, grammar  $G$ .



Given  $\beta$ , the r.h.s. of a production,  $TPL(\beta, l, Iparam, Conj)$  yields both a parameter list,  $Iparam$ , and a conjunction of predicates,  $Conj$ . More specifically  $TPL(\beta, n, Iparam, Conj)$  is true iff  $\beta = \beta_n B_n \beta_{n+1} \dots B_k \beta_{k+1} \in (V_N \cup V_T)^*$  where each  $\beta_i \in V_T^*$  and  $B_i \in V_N$  and  $Iparam = \beta_n x_n \beta_{n+1} \dots x_k \beta_{k+1}$ , and  $Conj = B_n(x_n) \wedge B_{n+1}(x_{n+1}) \wedge \dots B_k(x_k)$ .

$$TPL(\epsilon, n, \epsilon, true) \leftarrow$$

$$TPL(s \beta, n, s \ Iparam, Conj) \leftarrow T(s)$$

$$\wedge TPL(\beta, n, Iparam, Conj)$$

$$TPL(S \beta, n, X(n) \ Iparam, S \ (" \ X(n) \ ") \wedge \ " \ Conj) \leftarrow N(S)$$

$$\wedge TPL(\beta, n+1, Iparam, Conj)$$

$T(s)$  is true iff  $s$  is a terminal symbol.

$N(S)$  is true iff  $S$  is a nonterminal symbol.

$X(n)$  is the letter  $x$  subscripted with the value of  $n$ , and similarly for  $Y(n)$  which will be used later.

#### 4. Functional Interpretation of Coupled Grammars

Coupled grammars have an input grammar and an output grammar. As each production of the input grammar is used, an output is associated with it as specified by the corresponding production of the output grammar. This relation can be specified by a logic program as shown in Table 4.1 for coupled grammars  $B$  and  $U$  translating binary to unary notation.

$A = A \ 1$	$A = AA \ 1$	$A(x_1 \ 1, y_1 y_1 \ 1) \leftarrow A(x_1, y_1)$
$A = A \ 0$	$A = AA$	$A(x_1 \ 0, y_1 y_1) \leftarrow A(x_1, y_1)$
$A = 1$	$A = 1$	$A(1, 1) \leftarrow$
$A = 0$	$A = \epsilon$	$A(0, \epsilon) \leftarrow$
grammar $B$	grammar $U$	logic program LPBU

Figure 4.1: Coupled grammars  $B$  and  $U$  and their logic program representation LPBU.

It is obvious that the input grammar describes all binary strings, and the output grammar (ambiguously) describes all unary strings. The translation doubles the value of the output string for each shift in positional notation and adds a further "1" if the number is odd.

The logic program predicates have two parameters, one for input and one for output. For example, the interpretation of the clause

$$A(x_1 1, y_1 y_1 1) \leftarrow A(x_1, y_1)$$

says that if  $x_1$  is an acceptable string producing output  $y_1$ , then  $x_1 1$  is an acceptable string producing  $y_1 y_1 1$  as output.

##### 5. Translation of Coupled Grammars to Their Functional Interpretation

Given coupled grammars  $G_1$  and  $G_2$ , one can construct a program that will perform a functional interpretation on a string, using  $G_1$  as the input grammar and  $G_2$  as the output grammar.  $FI(G_1, G_2, LP)$  is true iff  $LP$ , a logic program, performs that interpretation. For example, from Figure 4.1,  $FI(B, U, LPBU)$ .

$$\begin{aligned} FI(\epsilon, \epsilon, \epsilon) &\leftarrow \\ FI((A = \beta) \cdot G_1, (A = \gamma) \cdot G_2, Z \cdot LP) &\leftarrow \\ &FI(G_1, G_2, LP) \\ &\wedge FIP(A, \beta, \gamma, Z) \end{aligned}$$

$FIP(A, \beta, \gamma, C)$  means that the functional interpretation of coupled productions  $A = \beta$  and  $A = \gamma$  is the logic procedure  $C$  where  $A$  is a single nonterminal and  $\beta$  and  $\gamma$  are strings in  $V^*$ . In the program below,  $Iparam$  and  $Oparam$  are each strings of variables and terminals and represent the input parameter and output parameter, respectively.  $R$  is a conjunction of predicates.

$$\begin{aligned} \text{FIP}(A, \beta, \gamma, A \text{ "(" Iparam "}, \text{" Oparam "}) &\leftarrow \text{" Conj"} \leftarrow \\ &\text{IN}(\beta, 1, \text{Iparam}, \text{Conj}) \\ &\wedge \text{OUT}(\text{Conj}, \gamma, \text{Oparam}) \end{aligned}$$

$\text{IN}(\beta, n, \text{Iparam}, \text{Conj})$  serves a similar function to  $\text{TPL}(\beta, n, \text{Iparam}, \text{Conj})$  except that we are producing two parameters instead of one to the predicates of  $\text{Conj}$ , i.e. whenever  $P(x_i)$  appears in  $\text{Conj}$  of  $\text{TPL}$ , then  $P(x_i, y_i)$  appears in  $\text{Conj}$  of  $\text{IN}$ .

$$\begin{aligned} \text{IN}(\varepsilon, n, \varepsilon, \text{true}) &\leftarrow \\ \text{IN}(s \ \beta, n, s \ \text{Iparam}, \text{Conj}) &\leftarrow \text{T}(s) \\ &\wedge \text{IN}(\beta, n, \text{Iparam}, \text{Conj}) \\ \text{IN}(S \ \beta, n, X(n) \ \text{Iparam}, S \text{ "(" } X(n) \text{ "}, \text{" Y(n) "}) &\wedge \text{" Conj"} \leftarrow \text{N}(S) \\ &\wedge \text{IN}(\beta, n+1, \text{Iparam}, \text{Conj}) \end{aligned}$$

$\text{OUT}(\text{Conj}, \gamma, \text{Oparam})$ :  $\text{Oparam}$  is the output parameter and if  $\gamma = \gamma_1 \gamma_2 \dots \gamma_n$  then  $\text{Oparam}$  denotes

$$\begin{aligned} s_1 s_2 \dots s_n \text{ where} \\ s_i &= \gamma_i \text{ if } t(\gamma_i) \\ &= y_j \text{ if } N(\gamma_i) \text{ and } j \text{ is the least integer such that} \\ &\quad \gamma_i(x_j, y_j) \text{ is a literal in Conj} \\ &= s_i \text{ if } N(\gamma_i) \text{ and } \gamma_i(x_j, y_j) \text{ is not in Conj} \end{aligned}$$

$$\begin{aligned} \text{OUT}(\text{Conj}, \varepsilon, \varepsilon) &\leftarrow \\ \text{OUT}(\text{Conj}, s \ \gamma, s \ \text{Oparam}) &\leftarrow \text{T}(s) \\ &\wedge \text{OUT}(\text{Conj}, \gamma, \text{Oparam}) \\ \text{OUT}(\text{Conj}, S \ \gamma, y \ \text{Oparam}) &\leftarrow \text{N}(S) \\ &\wedge \text{FIND}(S, \text{Conj}, y) \\ &\wedge \text{OUT}(\text{Conj}, \gamma, \text{Oparam}) \end{aligned}$$



FIND(S,Conj,z): For nonterminal S, and conjunct Conj,

$z = y$  for leftmost predicate in Conj that is of the form  $S(x,y)$   
 $= z$  if no such predicate exists in Conj

FIND(S,true,S)  $\leftarrow$

FIND(S,S "(" x "," y ")"  $\wedge$  " Conj,y)  $\leftarrow$

FIND(S,R "(" x "," y ")"  $\wedge$  " Conj,z)  $\leftarrow$  (S $\neq$ R)  
 $\wedge$  FIND(S,Conj,z)

For example, if A, B, C and D are nonterminals and # and ! are terminals, then the following are true:

FIP(A,#BCD,CB!, A(#x<sub>1</sub>x<sub>2</sub>x<sub>3</sub>,y<sub>2</sub>y<sub>1</sub>!)  $\leftarrow$  B(x<sub>1</sub>,y<sub>1</sub>)  $\wedge$  C(x<sub>2</sub>,y<sub>2</sub>)  $\wedge$  D(x<sub>3</sub>,y<sub>3</sub>))

IN(#BCD, 1, #x<sub>1</sub>x<sub>2</sub>x<sub>3</sub>, B(x<sub>1</sub>,y<sub>1</sub>)  $\wedge$  C(x<sub>2</sub>,y<sub>2</sub>)  $\wedge$  D(x<sub>3</sub>,y<sub>3</sub>))

OUT((B(x<sub>1</sub>,y<sub>1</sub>)  $\wedge$  C(x<sub>2</sub>,y<sub>2</sub>)  $\wedge$  D(x<sub>3</sub>,y<sub>3</sub>)), CB!, y<sub>2</sub>y<sub>1</sub>!)

## 6. Parse Sequence Interpretation of Coupled Grammars

Given a grammar G, with each string in L(G) is associated one (or more) parse sequences. A parse sequence is a sequence of integers corresponding to the production numbers as they are applied in a left-to-right parse.

Suppose we have two arbitrary coupled grammars, and each is used to parse a string in its language. The strings are defined to be equivalent if they have a parse sequence in common, as shown in Figure 6.1.

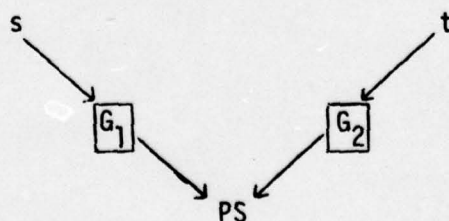


Figure 6.1: Parse sequence interpretation with the parse sequences not used intermediately.

Alternatively, take the same two coupled grammars, where the input grammar is used to generate a parse sequence and then that parse sequence is used with the output grammar to generate output as shown in Figure 6.2. We have then defined a means of translation.

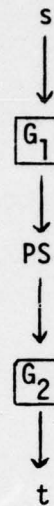


Figure 6.2: Parse sequence interpretation with the parse sequence used as an intermediate form.

$E = E + T$	$P = P P +$
$E = T$	$P = P$
$T = T * D$	$P = P P *$
$T = D$	$P = P$
$D = ( E )$	$P = P$
$D = 0$	$P = 0$
$D = 1$	$P = 1$
$D = 2$	$P = 2$
$D = 3$	$P = 3$
$\vdots$	$\vdots$
$D = 9$	$P = 9$
input grammar	output grammar

Table 6.1: Parse sequence coupled grammars.



The coupled grammars in Table 6.1 relate infix expressions and Polish expressions. The input grammar is unambiguous; every parse sequence from it is a parse sequence for the output grammar. Thus, there is a Polish form for every infix expression. Because of rules  $P = P$  in the output grammar, it is ambiguous. For every Polish string there are infinitely many parses which are also parses for the input grammar. Each defines a correct translation. There are also parses from the output grammar which are meaningless relative to the input grammar.

### 7. Translation of Coupled Grammars to Their Parse Sequence Interpretation

$PSI(G_1, G_2, s, t)$  is true iff grammar  $G_1$  parsing string  $s$  and grammar  $G_2$  parsing string  $t$  give the same parse sequence. We give two different definitions for  $PSI$ . The first generates an explicit parse sequence from an input string and its relevant grammar, then uses that sequence to generate a corresponding string in the language of the other grammar.

$$\begin{aligned} PSI(G_1, G_2, s, t) \leftarrow \\ & \text{PARSE}(G_1, s, PS) \\ & \wedge \text{GENERATE}(G_2, PS, t) \end{aligned}$$

$\text{PARSE}(G_1, s, PS)$  is true iff  $PS$  is a parse sequence of string  $s$  in grammar  $G_1$ . The parse is accomplished by creating an associated grammar  $GS$ , then using logic program  $FI$  to translate  $G_1$  and  $GS$  into a logic program that carries out a functional interpretation between the grammars, and, finally, to execute that logic program with string  $s$  as input to produce parse sequence  $PS$ .

$$\begin{aligned} \text{PARSE}(G_1, s, PS) \leftarrow \\ & \text{SEQ}(G_1, 1, GS) \\ & \wedge \text{FI}(G_1, GS, LP) \\ & \wedge \text{EXEC}((\text{"+" Start}(G_1) \text{"(" s "," PS ")"})) \\ & \cdot LP) \end{aligned}$$

$SEQ(G_1, n, GS)$  is true iff  $G_1$  and  $GS$  are context-free grammars,  $n$  is a positive integer, and by considering the productions of  $G_1$  sequenced starting at  $n$ ,  $GS$  consists of corresponding productions in which each r.h.s. is the non-terminals, in order, of its corresponding production in  $G_1$ , followed by the sequence number of that production.

Formally,  $SEQ$  is defined:

$$\begin{aligned} SEQ(\epsilon, n, \epsilon) &\leftarrow \\ SEQ((A = \alpha) \cdot G, n, (A = \beta n) \cdot GS) &\leftarrow \\ &SEQ(G, n+1, GS) \\ &\wedge STRIP(\alpha, \beta) \end{aligned}$$

$STRIP(\alpha, \beta)$  is true iff  $\alpha$  is a string in  $V^*$ , and  $\beta$  is  $\alpha$  with terminals stripped away.

$$\begin{aligned} STRIP(\epsilon, \epsilon) &\leftarrow \\ STRIP(s \alpha, \beta) &\leftarrow T(s) \\ &\wedge STRIP(\alpha, \beta) \\ STRIP(S \alpha, S \beta) &\leftarrow N(S) \\ &\wedge STRIP(\alpha, \beta) \end{aligned}$$

$EXEC(P)$  is a meta-procedure that executes logic program  $P$ . For an example of  $PARSE$ , consider

$G_1:$	$s:$
$E = E+T$	$a*a + a*a$
$E = T$	
$T = T*a$	
$T = a$	

the calls and computed values are:

SEQ( $G_1, 1, GS$ ) in which  $GS$  becomes:

$$E = ET1$$

$$E = T2$$

$$T = T3$$

$$T = 4$$

FI( $G_1, GS, LP$ ) in which  $LP$  becomes:

$$E(x_1 "+" x_2, y_1 y_2 \ 1) \leftarrow E(x_1, y_1) \wedge T(x_2, y_2)$$

$$E(x_1, y_1 \ 2) \leftarrow T(x_1, y_1)$$

$$T(x_1 "*" a, y_1 y_2 \ 3) \leftarrow T(x_1, y_1)$$

$$T(a, 4) \leftarrow$$

EXEC( $\leftarrow E("a*a+a*a", PS) \cdot LP$ ) in which  $PS$  becomes:

432431

Now this completes the first half of the definition of  $PSI$ . Given the parse sequence constructed in the above process we can use it to drive a right-most derivation in  $G_2$ , to create string  $t$ .

We now define the predicate  $GENERATE$ . We know intuitively that parsing and generation of strings are inverse operations. That would say that we could define  $GENERATE$  in terms of  $PARSE$ , thus:

$$GENERATE(G_2, PS, t) \leftrightarrow PARSE(G_2, t, PS)$$

The way we have used  $PARSE$  (and think of parsing) is that the string in the language is given and we generate the parse sequence as a side-effect of the recognition process. Suppose the parse sequence and the grammar are given. Can we use  $PARSE$  to create the input? A useful property of logic programs is that they describe truth about relationships, and while they can drive computations, the direction of the computation is usually arbitrary. Let's follow the computation to see if the string  $t$  can be appropriately computed.



Continuing the example above, let  $G_2$  be:

$$P = P P +$$

$$P = P$$

$$P = P P *$$

$$P = a$$

Then  $\text{PARSE}(G_2, t, 432431)$  calls  $\text{SEQ}(G_2, 1, GS')$  which produces  $GS'$ :

$$P = P P 1$$

$$P = P 2$$

$$P = P P 3$$

$$P = 4$$

Then  $\text{FI}(G_2, GS', LP')$  computes  $LP'$ :

$$P(x_1 x_2 +, y_1 y_2 1) \leftarrow P(x_1, y_1) \wedge P(x_2, y_2)$$

$$P(x_1, y_1 2) \leftarrow P(x_1, y_1)$$

$$P(x_1 x_2 *, y_1 y_2 3) \leftarrow P(x_1, y_1) \wedge P(x_2, y_2)$$

$$P(a, 4) \leftarrow$$

Executing  $LP'$  with call  $\leftarrow P(t, 432431)$ , we compute  $t = aa*aa*+$ , the desired answer.

Therefore, we could have defined  $PSI$  as:

$$PSI(G_1, G_2, s, t) \leftarrow$$

$$\text{PARSE}(G_1, s, PS)$$

$$\wedge \text{PARSE}(G_2, t, PS)$$

And, we see that our original claim of the equivalence of the two definitions is reflected in their having a single formal specification.

## 8. Mixed Grammars

Suppose we have a set of nonterminals  $V_N$  and two disjoint sets of terminals

$V_I$  and  $V_0$ , and a context-free grammar

$$[V_N, V_I \cup V_0, P, S] .$$

The productions in  $P$  are of the form

$$A = \alpha$$

where

$$\alpha \in (V_N \cup V_I \cup V_0)^*.$$

They are equivalent to the coupled grammars

$$[V_N, V_I, P_I, S] \quad \text{and} \quad [V_N, V_0, P_0, S]$$

where all elements of  $V_0$  are deleted from the productions of  $P$  to give  $P_I$  and vice-versa for  $P_0$ .

The grammars can be interpreted either as functionally coupled or parse sequence coupled. They are equivalent to the simple syntax-directed translation schemata of Aho and Ullman [1, Vol. I].

Notationally speaking, it is convenient for  $V_I \cap V_0$  to be nonempty, thus we establish the convention of double quotes delimiting the members of  $V_I$  (as in earlier sections of this paper) and single quotes delimiting the members of  $V_0$ .

$$E = E "+" T '+'$$

$$E = T$$

$$T = T "*" "a" 'a' '*'$$

$$T = "a" 'a'$$

Table 8.1: A mixed grammar describing the translation from infix to Polish.

For example, the mixed grammar in Table 8.1 has the same effect as the coupled grammars in the preceding examples. It will also accept other strings but its behavior is then of no interest.



The advantages of mixed grammars are that they are directly executable on a simple pushdown store machine, and that their notation makes implicit, unavoidable and natural the constraints for simple syntax-directed translation schemata. One can produce a logic program similar to PSI and FI to translate mixed grammars to logic.

## 9. Conclusions

We have established some relationships among context-free grammars, translation schemata, and logic. The interpretation of paired grammars has been extended in several ways. We have defined a set of translation programs that are actually sets of logic theorems. They are concise, and the correctness of each program can be established by proving each theorem individually. The process of parsing and the process of generation which are inverses are shown to have the same formal specification.

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